

# On positivity of quantum measure and of effective action in area tensor Regge calculus

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## Abstract

Because of unboundedness of the general relativity action, Euclidean version of the path integral in general relativity requires definition. Area tensor Regge calculus is considered in the representation with independent area tensor and finite rotation matrices. Being integrated over rotation matrices the path integral measure in area tensor Regge calculus is rewritten by moving integration contours to complex plane so that it looks as that one with effective action in the exponential with positive real part. We speculate that positivity of the measure can be expected in the most part of range of variation of area tensors.

PACS numbers: 04.60.-m Quantum gravity

The formal nonrenormalisability of quantum version of general relativity (GR) may cause us to try to find alternatives to the continuum description of underlying spacetime structure. An example of such the alternative description may be given by Regge calculus (RC) suggested in 1961 [1]. It is the exact GR developed in the piecewise flat spacetime which is a particular case of general Riemannian spacetime [2]. In turn, the general Riemannian spacetime can be considered as limiting case of the piecewise flat spacetime [3]. Any piecewise flat spacetime is simplicial one: it can be represented as collection of a (countable) number of the flat 4-dimensional *simplices*(tetrahedrons), and its geometry is completely specified by the countable number of the freely chosen lengths of all edges (or 1-simplices). Thus, RC implies a *discrete* description *alternative* to the usual continuum one. For a review of RC and alternative discrete gravity approaches see, e. g., [4].

The discrete nature of the simplicial description presents a difficulty in the (canonical) quantization of such the theory due to the absence of a regular continuous coordinate playing the role of time. Therefore one cannot immediately develop Hamiltonian formalism and canonical (Dirac) quantization. To do this we need to return to the partially continuum description, namely, with respect to only one direction shrinking sizes of all the simplices along this direction to those infinitely close to zero. The linklengths and other geometrical quantities become functions of the continuous coordinate taken along this direction. We can call this coordinate time  $t$  and develop quantization procedure with respect to this time. The result of this procedure can be formulated as some path integral measure. It is quite natural to consider this measure as a (appropriately defined) limiting continuous time form of a measure on the set of the original completely discrete simplicial spacetimes. This last completely discrete measure is just the object of interest to be found. The requirement for this measure to have the known limiting continuous time form can be considered as a starting postulate in our construction. The issuing principles are of course not unique, and another approaches to defining quantum measure in RC based on another physical principles do exist [5, 6].

The above condition for the completely discrete measure to possess required continuous time limit does not defines it uniquely as long as only one fixed direction which defines  $t$  is considered. However, different coordinate directions should be equivalent and we have a right to require for the measure to result in the canonical quantization

measure in the continuous time limit *whatever* coordinate direction is chosen to define a time. These requirements are on the contrary a priori too stringent, and it is important that on some configuration superspace (extended in comparison with superspace of the genuine simplicial geometries) such the measure turns out to exist.

Briefly speaking, we should, first, find continuous time limit for Regge action, recast it in the canonical Hamiltonian form and write out the Hamiltonian path integral, the measure in the latter being called for a moment the continuous time measure; second, we should check for existence and (if exists) find the measure obeying the property to tend in the continuous time limit (with concept "to tend" being properly defined) to the found continuous time measure irrespectively of the choice of the time coordinate direction. When passing to the continuous time RC we are faced with the difficulty that the description of the infinitely flattened in some direction simplex purely in terms of the lengths is singular.

The way to avoid singularities in the continuous time limit is to extend the set of variables via adding the new ones having the sense of angles and considered as independent variables. Such the variables are the finite rotation matrices which are the discrete analogs of the connections in the continuum GR. The situation considered is analogous to that one occurred when recasting the Einstein action in the Hilbert-Palatini form,

$$\frac{1}{2} \int R \sqrt{g} d^4x \Leftarrow \frac{1}{8} \int \epsilon_{abcd} \epsilon^{\lambda\mu\nu\rho} e_\lambda^a e_\mu^b [\partial_\nu + \omega_\nu, \partial_\rho + \omega_\rho]^{cd} d^4x, \quad (1)$$

where the tetrad  $e_\lambda^a$  and connection  $\omega_\lambda^{ab} = -\omega_\lambda^{ba}$  are independent variables, the RHS being reduced to LHS in terms of  $g_{\lambda\mu} = e_\lambda^a e_{a\mu}$  if we substitute for  $\omega_\lambda^{ab}$  solution of the equations of motion for these variables in terms of  $e_\lambda^a$ . The Latin indices  $a, b, c, \dots$  are the vector ones with respect to the local Euclidean frames which are introduced at each point  $x$ .

Now in RC the Einstein action in the LHS of (1) becomes the Regge action,

$$\sum_{\sigma^2} \alpha_{\sigma^2} |\sigma^2|, \quad (2)$$

where  $|\sigma^2|$  is the area of a triangle (the 2-simplex)  $\sigma^2$ ,  $\alpha_{\sigma^2}$  is the angle defect on this triangle, and summation run over all the 2-simplices  $\sigma^2$ . The discrete analogs of the tetrad and connection, edge vectors and finite rotation matrices, were first considered in [7]. The local Euclidean frames live in the 4-simplices now, and the analogs of the

connection are defined on the 3-simplices  $\sigma^3$  and are the matrices  $\Omega_{\sigma^3}$  connecting the frames of the pairs of the 4-simplices  $\sigma^4$  sharing the 3-faces  $\sigma^3$ . These matrices are the finite  $\text{SO}(4)$  rotations in the Euclidean case (or  $\text{SO}(3,1)$  rotations in the Lorentzian case) in contrast with the continuum connections  $\omega_\lambda^{ab}$  which are the elements of the Lee algebra  $\text{so}(4)(\text{so}(3,1))$  of this group. This definition includes pointing out the direction in which the connection  $\Omega_{\sigma^3}$  acts (and, correspondingly, the opposite direction, in which the  $\Omega_{\sigma^3}^{-1} = \bar{\Omega}_{\sigma^3}$  acts), that is, the connections  $\Omega$  are defined on the *oriented* 3-simplices  $\sigma^3$ . Instead of RHS of (1) we use exact representation which we suggest in our work [8],

$$S(v, \Omega) = \sum_{\sigma^2} |v_{\sigma^2}| \arcsin \frac{v_{\sigma^2} \circ R_{\sigma^2}(\Omega)}{|v_{\sigma^2}|} \quad (3)$$

where we have defined  $A \circ B = \frac{1}{2} A^{ab} B_{ab}$ ,  $|A| = (A \circ A)^{1/2}$  for the two tensors  $A, B$ ;  $v_{\sigma^2}$  is the dual bivector of the triangle  $\sigma^2$  in terms of the vectors of its edges  $l_1^a, l_2^a$ ,

$$v_{\sigma^2 ab} = \frac{1}{2} \epsilon_{abcd} l_1^c l_2^d \quad (4)$$

(in some 4-simplex frame containing  $\sigma^2$ ). The curvature matrix  $R_{\sigma^2}$  on the 2-simplex  $\sigma^2$  is the path ordered product of the connections  $\Omega_{\sigma^3}^{\pm 1}$  on the 3-simplices  $\sigma^3$  sharing  $\sigma^2$  along the contour enclosing  $\sigma^2$  once and contained in the 4-simplices sharing  $\sigma^2$ ,

$$R_{\sigma^2} = \prod_{\sigma^3 \supset \sigma^2} \Omega_{\sigma^3}^{\pm 1}. \quad (5)$$

As we can show, when substituting as  $\Omega_{\sigma^3}$  the genuine rotations connecting the neighbouring local frames as functions of the genuine Regge lengths into the equations of motion for  $\Omega_{\sigma^3}$  with the action (3) we get exactly the closure condition for the surface of the 3-simplex  $\sigma^3$  (vanishing the sum of the bivectors of its 2-faces) written in the frame of one of the 4-simplices containing  $\sigma^3$ , that is, the identity. This means that (3) is the exact representation for (2). At the same time, general solution to the equations of motion is wider than that leading to  $R_{\sigma^2}(\Omega)$  rotating around  $\sigma^2$  by the defect angle  $\alpha_{\sigma^2}$ .

We can pass to the continuous time limit in (3) in a nonsingular manner and recast it to the canonical (Hamiltonian) form [9]. This allows us to write out Hamiltonian path integral. The above problem of finding the measure which results in the Hamiltonian path integral measure in the continuous time limit whatever coordinate is chosen as time has solution in 3 dimensions [10]. A specific feature of the 3D case important for that is commutativity of the dynamical constraints leading to a simple form of

the functional integral. The 3D action looks like (3) with area tensors  $v_{\sigma^2}$  substituted by the egde vectors  $\mathbf{l}_{\sigma^1}$  independent of each other. In 4 dimensions, the variables  $v_{\sigma^2}$  are not independent but obey a set of (bilinear) *intersection relations*. For example, tensors of the two triangles  $\sigma_1^2, \sigma_2^2$  sharing an edge satisfy the relation

$$\epsilon_{abcd} v_{\sigma_1^2}^{ab} v_{\sigma_2^2}^{cd} = 0. \quad (6)$$

These purely geometrical relations can be called kinematical constraints. The idea is to construct quantum measure first for the system with formally independent area tensors. That is, originally we concentrate on quantization of the dynamics while kinematical relations of the type (6) are taken into account at the second stage. Note that the RC with formally independent (scalar) areas have been considered in the literature [4, 11].

The theory with formally independent area tensors can be called area tensor RC. Consider the Euclidean case. The Einstein action is not bounded from below, therefore the Euclidean path integral itself requires careful definition. Our result for the constructed in the above way completely discrete quantum measure [12] can be written as a result for vacuum expectations of the functions of the field variables  $v, \Omega$ . Upon passing to integration over imaginary areas with the help of the formal replacement of the tensors of a certain subset of areas  $\pi$  over which integration in the path integral is to be performed,

$$\pi \rightarrow -i\pi,$$

the result reads

$$\begin{aligned} \langle \Psi(\{\pi\}, \{\Omega\}) \rangle &= \int \Psi(-i\{\pi\}, \{\Omega\}) \exp \left( - \sum_{\substack{t\text{-like} \\ \sigma^2}} \tau_{\sigma^2} \circ R_{\sigma^2}(\Omega) \right) \\ &\cdot \exp \left( i \sum_{\substack{\text{not} \\ t\text{-like} \\ \sigma^2}} \pi_{\sigma^2} \circ R_{\sigma^2}(\Omega) \right) \prod_{\substack{\text{not} \\ t\text{-like} \\ \sigma^2}} d^6 \pi_{\sigma^2} \prod_{\sigma^3} \mathcal{D}\Omega_{\sigma^3} \\ &\equiv \int \Psi(-i\{\pi\}, \{\Omega\}) d\mu_{\text{area}}(-i\{\pi\}, \{\Omega\}), \end{aligned} \quad (7)$$

where  $\mathcal{D}\Omega_{\sigma^3}$  is the Haar measure on the group  $\text{SO}(4)$  of connection matrices  $\Omega_{\sigma^3}$ . Appearance of some set  $\mathcal{F}$  of triangles  $\sigma^2$  integration over area tensors of which is omitted (denoted as "t-like" in (7)) is connected with that integration over *all* area tensors is generally infinite, in particular, when normalizing measure (finding  $\langle 1 \rangle$ ). Indeed, different  $R_{\sigma^2}$  for  $\sigma^2$  meeting at a given link  $\sigma^1$  are connected by Bianchi identities [1].

Therefore for the spacetime of Minkowsky signature (when exponent is oscillating over all the area tensors) the product of  $\delta^6(R_{\sigma^2} - \bar{R}_{\sigma^2})$  for all these  $\sigma^2$  which follow upon integration over area tensors for these  $\sigma^2$  contains singularity of the type of  $\delta$ -function squared. To avoid this singularity we should confine ourselves by only integration over area tensors on those  $\sigma^2$  on which  $R_{\sigma^2}$  are independent, and complement  $\mathcal{F}$  to this set of  $\sigma^2$  are those  $\sigma^2$  on which  $R_{\sigma^2}$  are by means of the Bianchi identities functions of these independent  $R_{\sigma^2}$ . Let us adopt regular way of constructing 4D simplicial structure of the 3D simplicial geometries (leaves) of the same structure. Denote by  $A, B, C, \dots$  vertices of the 4D simplicial complex while  $n$ -simplex  $\sigma^n$  is denoted by the set of its  $n + 1$  vertices in round brackets (unordered sequence),  $(A_1 A_2 \dots)$ . The  $i, k, l, \dots$  are vertices of the current leaf,  $i^+, k^+, l^+, \dots$  and  $i^-, k^-, l^-, \dots$  are corresponding vertices of the nearest future and past in  $t$  leaves. Or, dealing with Euclidean time, we shall speak of the "upper" and "lower" leaves, respectively. Each vertex is connected by links (edges) with its  $\pm$ -images. These links (of the type of  $(ii^+)$ ,  $(ii^-)$ ) will be called *t-like* ones (do not mix with the term "timelike" which is reserved for the local frame components). The *leaf* links  $(ik)$  are completely contained in the 3D leaf. There may be *diagonal* links  $(ik^+)$ ,  $(ik^-)$  connecting a vertex with the  $\pm$ -images of its neighbors. We call arbitrary simplex *t-like* one if it has *t-like* edge, the *leaf* one if it is completely contained in the 3D leaf and *diagonal* one in other cases. It can be seen that the set of the *t-like* triangles is fit for the role of the above set  $\mathcal{F}$ . In the case of general 4D simplicial structure we can deduce that the set  $\mathcal{F}$  of the triangles with the Bianchi-dependent curvatures pick out some one-dimensional field of links, and we can simply take it as definition of the coordinate  $t$  direction so that  $\mathcal{F}$  be just the set of the *t-like* triangles. Also existence of the set  $\mathcal{F}$  naturally fits our initial requirement that limiting form of the full discrete measure when any one of the coordinates (not necessarily  $t$ !) is made continuous by flattening the 4-simplices in the corresponding direction should coincide with Hamiltonian path integral (with that coordinate playing the role of time). Namely, in the Hamiltonian formalism absence of integration over area tensors of triangles which pick out some coordinate  $t$  (*t-like* ones) corresponds to some gauge fixing.

There is the invariant (Haar) measure  $\mathcal{D}\Omega$  in (7) which looks natural from symmetry considerations. From the formal point of view, in the Hamiltonian formalism (when one of the coordinates is made continuous) this arises when we write out standard

Hamiltonian path integral for the Lagrangian with the kinetic term  $\pi_{\sigma^2} \circ \bar{\Omega}_{\sigma^2} \dot{\Omega}_{\sigma^2}$  [10, 12]. To this end, one might pass to the variables  $\Omega_{\sigma^2} \pi_{\sigma^2} = P_{\sigma^2}$  and  $\Omega_{\sigma^2}$  (in 3D case used in [14, 10]). The kinetic term  $P\dot{\Omega}$  with arbitrary matrices  $P, \Omega$  leads to the standard measure  $d^{16}P d^{16}\Omega$ , but there are also  $\delta$ -functions taking into account II class constraints to which  $P, \Omega$  are subject,  $\delta^{10}(\bar{\Omega}\Omega - 1)\delta^{10}(\bar{\Omega}P + \bar{P}\Omega)$ . Integrating out these just gives  $d^6\pi \mathcal{D}^6\Omega$ . Following our strategy of recovering full discrete measure from requirement that it reduces to the Hamiltonian path integral whatever coordinate is made continuous, the same Haar measure should be present also in the full discrete measure.

One else specific feature of the quantum measure is the absence of the inverse trigonometric function 'arcsin' in the exponential, whereas the Regge action (3) contains such functions. This is connected with using the canonical quantization at the intermediate stage of derivation: in gravity this quantization is completely defined by the constraints, the latter being equivalent to those ones without arcsin (in some sense on-shell).

In what follows, it is convenient to split antisymmetric matrices ( $\pi$  and generator of  $R$ ) into self- and antiselfdual parts, then the measure (7) splits into two factors, in the self- and antiselfdual sectors,

$$\pi_{ab} \equiv \frac{1}{2} {}^+\pi_k {}^+\Sigma_{ab}^k + \frac{1}{2} {}^-\pi_k {}^-\Sigma_{ab}^k \quad (8)$$

$${}^\pm R = \exp({}^\pm\phi {}^\pm\Sigma) = \cos {}^\pm\phi + {}^\pm\Sigma {}^\pm\mathbf{n} \sin {}^\pm\phi \quad (9)$$

$$d\mu_{\text{area}} = d{}^+\mu_{\text{area}} d{}^-\mu_{\text{area}}. \quad (10)$$

Here  ${}^\pm\mathbf{n} = {}^\pm\phi / {}^\pm\phi$  is unit vector and the basis of self- and antiselfdual matrices  $i {}^\pm\Sigma_{ab}^k$  obeys the Pauli matrix algebra.

Since as pointed out below the eqs. (3) - (5) the classical equations of motion for  $\Omega$  do not restrict the resulting  $R_{\sigma^2}(\Omega)$  be exactly the rotation around  $\sigma^2$  by the defect angle  $\alpha_{\sigma^2}$ , the sense of  $\Omega$ ,  $R(\Omega)$  as physical observables is restricted. Consider averaging functions of only area tensors  $\pi_{\sigma^2}$ . By the properties of invariant measure, integrations over  $\prod \mathcal{D}\Omega_{\sigma^3}$  in (7) reduce to integrations over  $\prod \mathcal{D}R_{\sigma^2}$  with independent  $R_{\sigma^2}$  (i. e.  $\sigma^2$  are just not  $t$ -like) and some number of connections  $\prod \mathcal{D}\Omega_{\sigma^3}$  which we can call gauge ones. The expectation value of any field monomial,  $\langle \pi_{\sigma_1^2}^{a_1 b_1} \dots \pi_{\sigma_n^2}^{a_n b_n} \rangle$  reduces to the (derivatives of)  $\delta$ -functions  $\delta(R_{\sigma_i^2}^{a_i b_i} - R_{\sigma_i^2}^{b_i a_i})$  which are then integrated out over  $\mathcal{D}R_{\sigma_i^2}$  giving finite nonzero answer. This is consequence of i) the underlying Dirac-Hamilton

principle of quantization (leading to  $d^6\pi_{\sigma^2}\mathcal{D}R_{\sigma^2}$  in the measure) and of ii) conception of independent area tensors (integrations over  $d^6\pi_{\sigma^2}$  are independent leading to  $\delta$ -functions). This holds in the Minkowsky spacetime as well (and in the first instance since oscillating exponent is present there from the very beginning). The Euclidean expectations values correspond to the Minkowskian ones in the spacelike region. The formal passing to the Euclidean version by simply writing  $\exp(-\pi_{\sigma^2} \circ R_{\sigma^2})$  in the measure (not with additional substitution  $\pi_{\sigma^2} \rightarrow -i\pi_{\sigma^2}$  in the integration variables as in (7)) might result, upon integrating over  $\mathcal{D}R_{\sigma^2}$ , in appearance of the terms with both factors,  $\exp(+|\pm\pi_{\sigma^2}|)$  and  $\exp(-|\pm\pi_{\sigma^2}|)$ . This is consequence of that iii)  $R_{\sigma^2}$  are *finite* SO(4) rotations, not elements of Lee group so(4) - therefore the stationary phase points in the integrals over  $\mathcal{D}R_{\sigma^2}$  correspond just to  $\pm\pi_{\sigma^2} \circ \pm R_{\sigma^2} = +|\pm\pi_{\sigma^2}|$  and  $\pm\pi_{\sigma^2} \circ \pm R_{\sigma^2} = -|\pm\pi_{\sigma^2}|$ . Due to the above mentioned finiteness of area monomial VEVs the growing exponents  $\exp(+|\pm\pi_{\sigma^2}|)$  should be excluded. Thus, the measure upon integration over connections should exponentially decrease with areas as  $\exp(-|\pm\pi_{\sigma^2}|)$ . Once again, collect the reasons for that,

- i) Dirac-Hamiltonian canonical quantization;
- ii) conception of independent area tensors;
- iii) connection matrices being finite SO(4) rotations, not elements of the Lee group so(4).

The definition of the Euclidean version (7) via  $\pi_{\sigma^2} \rightarrow -i\pi_{\sigma^2}$ , as well as of Minkowski-an one, contains oscillating exponent. It is possible to reproduce the results of above considered calculation of area monomial VEVs through  $\delta$ -functions of (antisymmetric part of) the curvature by integrating monomials with monotonic exponent in terms of genuine  $\pi_{\sigma^2}$  by moving integration contour over curvature to complex plane [15]. This contour should start at  $\pm\pi_{\sigma^2} \circ \pm R_{\sigma^2} = +|\pm\pi_{\sigma^2}|$ , not at  $\pm\pi_{\sigma^2} \circ \pm R_{\sigma^2} = -|\pm\pi_{\sigma^2}|$ . If  $\pm R$  appears in the exponential in the form  $\pm\pi \circ \pm R$ , then appropriate complex change of variable  $\pm\phi$  parameterizing  $\pm R$  corresponds to

$$\pm\phi = \frac{\pi}{2} + i\pm\eta, \quad -\infty < \pm\eta < +\infty, \quad (11)$$

$$\pm\theta = i\pm\zeta, \quad 0 \leq \pm\zeta < +\infty \quad (12)$$

where  $\pm\theta$  is the azimuthal angle of  $\pm\phi$  w.r.t.  $\pm\pi$ , the polar angle  $\pm\chi$  remaining the same.

Now generalize (11), (12) to the case when  $\pm R$  enters in the form  $\pm m \circ \pm R$  where



${}^\pm m$  has not only antisymmetric, but also scalar part,

$${}^\pm m = \frac{1}{2} {}^\pm \mathbf{m} \cdot {}^\pm \Sigma + \frac{1}{2} {}^\pm m_0 \cdot 1. \quad (13)$$

Of course, in this case  ${}^\pm m$  can not be (anti)selfdual part of anything nor (anti)selfdual matrix itself. Here index  $\pm$  means simply that it is sum of products of (anti)selfdual matrices. The latter arise when we express curvatures on  $t$ -like triangles in terms of independent ones with the help of Bianchi identities. These curvatures can depend on the given  $R_{\sigma^2}^{\pm 1}$  linearly or not depend at all. Therefore  ${}^\pm m_{\sigma^2} \circ {}^\pm R_{\sigma^2}$  is the general form of dependence on the given  ${}^\pm R_{\sigma^2}$  in the exponential of (7). General form of integral over given curvature matrix is

$$\int \exp(-{}^\pm \mathbf{m} {}^\pm \mathbf{n} \sin {}^\pm \phi - {}^\pm m_0 \cos {}^\pm \phi) \frac{\sin^2 {}^\pm \phi}{{}^\pm \phi^2} d^3 {}^\pm \phi, \quad (14)$$

where, remind,  ${}^\pm \mathbf{n} = {}^\pm \phi / {}^\pm \phi$ , and azimuthal angle of  ${}^\pm \phi$  w.r.t.  ${}^\pm \mathbf{m}$  is  ${}^\pm \theta$ . Apply (11) and then (12) to the *shifted*  ${}^\pm \phi$ ,

$${}^\pm \theta = i {}^\pm \zeta, \quad {}^\pm \phi + {}^\pm \alpha = \frac{\pi}{2} + i {}^\pm \eta, \quad (15)$$

$$\cos {}^\pm \alpha = \frac{\sqrt{{}^\pm \mathbf{m}^2} \cosh {}^\pm \zeta}{\sqrt{{}^\pm \mathbf{m}^2 \cosh^2 {}^\pm \zeta + {}^\pm m_0^2}}, \quad \sin {}^\pm \alpha = \frac{{}^\pm m_0}{\sqrt{{}^\pm \mathbf{m}^2 \cosh^2 {}^\pm \zeta + {}^\pm m_0^2}}. \quad (16)$$

The general case of complex  ${}^\pm \mathbf{m}$ ,  ${}^\pm m_0$  is implied. Important is that  ${}^\pm \Sigma_k$  are real-valued so that orthogonal conjugation operation is commuting with analytic continuation. The branch of the function  $\sqrt{z}$  is chosen in the complex plane of  $z$  with cut along negative real half-axis such that  $\sqrt{1} = 1$ . (In particular, this means that  $\Re \sqrt{z} \geq 0$ .) The integral over  ${}^\pm \eta$ ,  ${}^\pm \zeta$  transforms to give

$$\int \exp(-{}^\pm m \circ {}^\pm R) \frac{\sin^2 {}^\pm \phi}{{}^\pm \phi^2} d^3 {}^\pm \phi = \frac{4\pi}{\sqrt{\text{tr } {}^\pm \bar{m} {}^\pm m}} K_1 \left( \sqrt{\text{tr } {}^\pm \bar{m} {}^\pm m} \right). \quad (17)$$

The  $K_1$  is the modified Bessel function.

The idea is to try to find some set of the 2-simplices  $\mathcal{M}$  so that exponential in (7) be representable in the form

$$- \sum_{\sigma^2 \in \mathcal{M}} m_{\sigma^2} \circ R_{\sigma^2} - \sum_{\sigma^2 \notin \mathcal{M}} \pi_{\sigma^2} \circ R_{\sigma^2} \quad (18)$$

where  $m_{\sigma^2} = \pi_{\sigma^2} +$  (linear in  $\{\tau_{\sigma^2}\}$  terms). The notation  $\{\dots\}$  means "the set of  $\dots$ ". The set  $\{m_{\sigma^2}\}$  depend on  $\{\tau_{\sigma^2}\}$  and on  $\{R_{\sigma^2} | \sigma^2 \notin \mathcal{M}\}$ , but not on  $\{R_{\sigma^2} | \sigma^2 \in \mathcal{M}\}$ .

Then integrations over  $\{R_{\sigma^2}|\sigma^2 \in \mathcal{M}\}$  can be explicitly performed according to eq. (17) giving

$$\begin{aligned}
d^{\pm}\mu_{\text{area}} &\equiv d^{\pm}\mathcal{N} \prod_{\substack{\text{not} \\ t\text{-like} \\ \sigma^2}} d^3{}^{\pm}\boldsymbol{\pi}_{\sigma^2}, \\
d^{\pm}\mathcal{N} &\Rightarrow \left[ \prod_{\sigma^2 \in \mathcal{M}} \frac{K_1\left(\sqrt{\text{tr}^{\pm}\bar{m}_{\sigma^2}{}^{\pm}m_{\sigma^2}}\right)}{\sqrt{\text{tr}^{\pm}\bar{m}_{\sigma^2}{}^{\pm}m_{\sigma^2}}} \right] \exp \left( - \sum_{\substack{\text{not} \\ t\text{-like} \\ \sigma^2 \notin \mathcal{M}}} |^{\pm}\boldsymbol{\pi}_{\sigma^2}| \cosh{}^{\pm}\zeta_{\sigma^2} \cosh{}^{\pm}\eta_{\sigma^2} \right) \\
&\cdot \prod_{\substack{\text{not} \\ t\text{-like} \\ \sigma^2 \notin \mathcal{M}}} \cosh^2{}^{\pm}\eta_{\sigma^2} d \cosh{}^{\pm}\eta_{\sigma^2} d \cosh{}^{\pm}\zeta_{\sigma^2} d{}^{\pm}\chi_{\sigma^2}
\end{aligned} \tag{19}$$

where  $\{m_{\sigma^2}|\sigma^2 \in \mathcal{M}\}$  depend on  $\{^{\pm}\eta_{\sigma^2}, {}^{\pm}\zeta_{\sigma^2}, {}^{\pm}\chi_{\sigma^2}|\sigma^2 \notin \mathcal{M}\}$  through  $R_{\sigma^2}$  parameterized by these,

$$\begin{aligned}
{}^{\pm}R_{\sigma^2} &= -i \sinh{}^{\pm}\eta_{\sigma^2} + {}^{\pm}\boldsymbol{\Sigma} \cdot {}^{\pm}\boldsymbol{n}_{\sigma^2} \cosh{}^{\pm}\eta_{\sigma^2}, \\
{}^{\pm}\boldsymbol{n}_{\sigma^2} &= \frac{{}^{\pm}\boldsymbol{\pi}_{\sigma^2}}{|^{\pm}\boldsymbol{\pi}_{\sigma^2}|} \cosh{}^{\pm}\zeta_{\sigma^2} + i(\sinh{}^{\pm}\zeta_{\sigma^2})({}^{\pm}\boldsymbol{e}_{1\sigma^2} \cos{}^{\pm}\chi_{\sigma^2} + {}^{\pm}\boldsymbol{e}_{2\sigma^2} \sin{}^{\pm}\chi_{\sigma^2})
\end{aligned} \tag{20}$$

where  ${}^{\pm}\boldsymbol{e}_{1\sigma^2}, {}^{\pm}\boldsymbol{e}_{2\sigma^2}$  together with  ${}^{\pm}\boldsymbol{\pi}_{\sigma^2}/|^{\pm}\boldsymbol{\pi}_{\sigma^2}|$  form orthonormal triple.

Rewrite (19) as

$$\begin{aligned}
d^{\pm}\mathcal{N} &\Rightarrow \exp \left( - \sum_{\sigma^2 \in \mathcal{M}} \sqrt{\text{tr}^{\pm}\bar{m}_{\sigma^2}{}^{\pm}m_{\sigma^2}} \cosh{}^{\pm}\zeta_{\sigma^2} \cosh{}^{\pm}\eta_{\sigma^2} \right. \\
&\left. - \sum_{\substack{\text{not} \\ t\text{-like} \\ \sigma^2 \notin \mathcal{M}}} |^{\pm}\boldsymbol{\pi}_{\sigma^2}| \cosh{}^{\pm}\zeta_{\sigma^2} \cosh{}^{\pm}\eta_{\sigma^2} \right) \prod_{\substack{\text{not} \\ t\text{-like} \\ \sigma^2}} \cosh^2{}^{\pm}\eta_{\sigma^2} d \cosh{}^{\pm}\eta_{\sigma^2} d \cosh{}^{\pm}\zeta_{\sigma^2} d{}^{\pm}\chi_{\sigma^2}
\end{aligned} \tag{21}$$

where abstract dummy variables  $\{^{\pm}\eta_{\sigma^2}, {}^{\pm}\zeta_{\sigma^2}, {}^{\pm}\chi_{\sigma^2}|\sigma^2 \in \mathcal{M}\}$  and integrations over them are introduced to represent  $K_1$  differently from what is given by equation (17) read from right to left. Remarkable is that it looks as path integral measure with *positive* (real part of) effective action whereas general relativity action remains unbounded from below upon formal Wick rotation. The price is that exponential in (21) has imaginary part, and positivity of the Euclidean measure (upon integrating out curvature matrices) does not follow automatically as in the case of the usual field theory with bounded action since explicitly real form of (21) reads

$$d^{\pm}\mathcal{N} \Rightarrow \exp \left( - \sum_{\sigma^2 \in \mathcal{M}} \Re \sqrt{\text{tr}^{\pm}\bar{m}_{\sigma^2}{}^{\pm}m_{\sigma^2}} \cosh{}^{\pm}\zeta_{\sigma^2} \cosh{}^{\pm}\eta_{\sigma^2} \right)$$

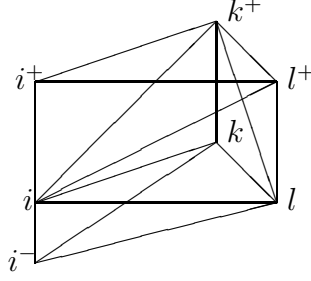
$$\begin{aligned}
& - \sum_{\substack{\text{not} \\ t\text{-like} \\ \sigma^2 \notin \mathcal{M}}} |{}^\pm \pi_{\sigma^2}| \cosh {}^\pm \zeta_{\sigma^2} \cosh {}^\pm \eta_{\sigma^2} \bigg) \cos \left( \sum_{\sigma^2 \in \mathcal{M}} \Im \sqrt{\text{tr } {}^\pm \bar{m}_{\sigma^2} {}^\pm m_{\sigma^2}} \cosh {}^\pm \zeta_{\sigma^2} \cosh {}^\pm \eta_{\sigma^2} \right) \\
& \cdot \prod_{\substack{\text{not} \\ t\text{-like} \\ \sigma^2}} \cosh^2 {}^\pm \eta_{\sigma^2} d \cosh {}^\pm \eta_{\sigma^2} d \cosh {}^\pm \zeta_{\sigma^2} d {}^\pm \chi_{\sigma^2}
\end{aligned} \tag{22}$$

that is nonconstant in sign due to cosine. Below we speculate that positivity should be expected in the most part of (if not in the whole) range of variation of area tensors  $\pi_{\sigma^2}$  if  $\tau_{\sigma^2}$  are sufficiently small.

To construct the set  $\mathcal{M}$ , note that due to the Bianchi identities dependence on the matrix  $R_{\sigma^2}$  on the given leaf/diagonal triangle  $\sigma^2$  in the exponential of (7) comes from all the triangles constituting together with this  $\sigma^2$  a closed surface. This is surface of the  $t$ -like 3-prism, one base of which is just the given  $\sigma^2$ , the lateral surface consists of  $t$ -like triangles and goes to infinity. In practice, replace this infinity by some lowest (initial) leaf where another base  $\sigma_0^2$  is located the tensor of which  $\pi_{\sigma_0^2}$  is taken as boundary value. Consider a variety of such prisms with upper bases  $\sigma^2$  placed in the uppest (final) leaf such that any link in this leaf belongs to one and only one of these bases. That is, lateral surfaces of different prisms do not have common triangles. Then the terms  $m_{\sigma^2} \circ R_{\sigma^2}$  in (18) represent contribution from these prisms,  $\mathcal{M}$  being the set of their bases in the uppest leaf.

To really reduce the measure to such form, we should express the curvature matrices on the  $t$ -like triangles in terms of those on the leaf/diagonal ones. The curvature on a leaf/diagonal triangle  $\sigma^2$  as product of  $\Omega$ s includes the two matrices  $\Omega$  on the  $t$ -like tetrahedrons  $\sigma^3$  adjacent to  $\sigma^2$  from above and from below. Knowing curvatures on the set of leaf/diagonal triangles inside any  $t$ -like 3-prism allows to successively express matrix  $\Omega$  on any  $t$ -like tetrahedron inside the prism in terms of matrix  $\Omega$  on the uppest  $t$ -like tetrahedron in this prism taken as boundary value. Expressions for the considered curvatures look like (fig.1)

$$\begin{aligned}
& \dots\dots\dots \\
& R_{(ikl)} = \dots \bar{\Omega}_{(i^{-}ikl)} \dots \Omega_{(ik^{+}kl)} \dots \\
& R_{(ik^{+})} = \dots \bar{\Omega}_{(ik^{+}kl)} \dots \Omega_{(ik^{+}l^{+})} \dots \\
& R_{(ik^{+}l^{+})} = \dots \bar{\Omega}_{(ik^{+}l^{+})} \dots \Omega_{(i^{+}ik^{+}l^{+})} \dots \\
& \dots\dots\dots
\end{aligned} \tag{23}$$

Figure 1: Fragment of the  $t$ -like 3-prism.

The dots in expressions for  $R$  mean matrices  $\Omega$  on the leaf/diagonal tetrahedrons which can be considered as gauge ones. We can step-by-step express  $\Omega_{(i-ikl)} \rightarrow \Omega_{(ik+k l)} \rightarrow \Omega_{(ik+l l)} \rightarrow \Omega_{(i+ik+l+)} \rightarrow \dots$  where the arrow means "in terms of". Knowing  $\Omega$ s on  $t$ -like tetrahedrons we can find the curvatures on  $t$ -like triangles, the products of these  $\Omega$ s,

$$R_{(i+ikl)} = \Omega_{(i+ikl_n)}^{\epsilon_{(ikl_n)l_{n-1}}} \dots \Omega_{(i+ikl_1)}^{\epsilon_{(ikl_1)l_n}}. \quad (24)$$

Here  $\epsilon_{(ikl)m} = \pm 1$  is some sign function. Thereby we find contribution of the  $t$ -like triangles in terms of independent curvature matrices (on the leaf/diagonal triangles).

In the continuum path integral formalism, one usually imposes boundary (initial/final) conditions to unambiguously define the measure. Consideration of the two previous paragraphs says that in our case fixing the initial leaf area tensors  $\pi_{\sigma_0^2}$  and final connections on the  $t$ -like tetrahedrons is appropriate. Thereby, in particular, non-trivial integrations reduce to those over matching each other sets  $d^6\pi_{\sigma^2}$  and  $\mathcal{D}^6 R_{\sigma^2}$  on the leaf/diagonal  $\sigma^2$ .

Note an important particular case when integrations in (19) are made over the whole sets  $\{\pi_{\sigma^2} | \sigma^2 \notin \mathcal{M}\}$  and  $\{R_{\sigma^2} | \sigma^2 \text{ is not } t\text{-like}\}$ . The resulting measure factorizes over the 3-prisms with upper bases constituting  $\mathcal{M}$ ,

$$d^{\pm} \mu_{\text{area}} \implies \prod_{\sigma^2 \in \mathcal{M}} \frac{K_1 \left( \sqrt{\text{tr } \pm \bar{m}_{\sigma^2} \pm m_{\sigma^2}} \right)}{\sqrt{\text{tr } \pm \bar{m}_{\sigma^2} \pm m_{\sigma^2}}} d^3 \pm \pi_{\sigma^2}. \quad (25)$$

Here  $m_{\sigma^2}$  is taken at  $\{R_{\sigma^2} = 0 | \text{not } t\text{-like } \sigma^2 \notin \mathcal{M}\}$  and differs from  $\pi_{\sigma^2}$  by a constant,  $m_{\sigma^2} = \pi_{\sigma^2} - \pi_{\sigma^2}^{(0)}$ . In turn,  $\pi_{\sigma^2}^{(0)}$  differs from the initial area tensor  $\pi_{\sigma_0^2}$  by the lateral 3-prism surface tensor:  $\pi_{\sigma^2}^{(0)} - \pi_{\sigma_0^2}$  is algebraic sum of tensors  $\tau_{\sigma^2}$  for  $\sigma^2$  constituting the lateral surface. The  $\pi_{\sigma^2}^{(0)}$  has geometrical meaning of *expected* value of area tensor  $\pi_{\sigma^2}$  when the surface of the 3-prism closes due to the (classical) equations of motion. The measure (25) describes quantum fluctuation of  $\pi_{\sigma^2}$  around  $\pi_{\sigma^2}^{(0)}$ . The (25) is explicitly positive.

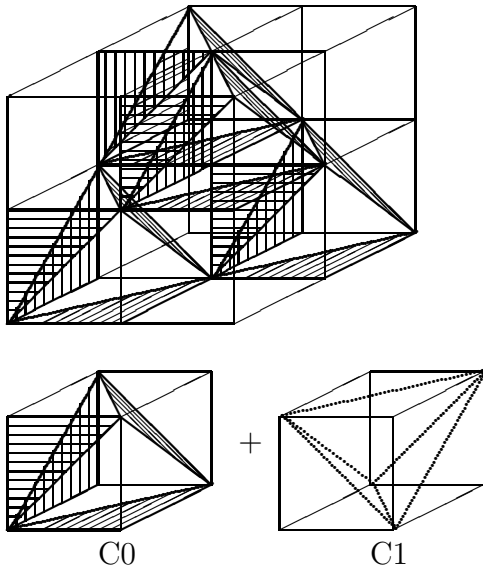


Figure 2: Periodic cell of the simplicial manifold with triangles marked (shaded) in such a way that any edge does belong to one and only one of marked triangles. It consists of  $2 \times 2 \times 2$  building blocks of the two types C0, C1 alternating in all three directions.

Thus, to represent exponential in (7) in the form (18) sufficient is to divide the whole set of links in the uppest 3D leaf into triples forming the triangles and take this set of triangles as  $\mathcal{M}$  in (18). It is clear that such set  $\mathcal{M}$  does exist not for an arbitrary 3D leaf (at least the number of links should be multiple of 3). In fig.2 probably the simplest periodic cell of simplicial lattice is shown where the set  $\mathcal{M}$  (shaded triangles) is also periodic.

Genuine simplicial decomposition possesses quite complex combinatorics, so let us demonstrate main features of the result of above calculation by using as example the cubic decomposition. The latter can be viewed as *sub*-minisuperspace of simplicial system if one starts from the simplest periodic simplicial complex with elementary 4-cubic cell divided by diagonals emitted from one of its vertices into 24 4-simplices [16]. Each 3-cube face built on three coordinate directions is divided into 6 tetrahedrons, and we simply put  $\Omega$ s on these tetrahedrons to be the same on the whole 3-cube. There are also the 3-cube faces built on two coordinate and one diagonal direction, and we put  $\Omega$ s on the tetrahedrons forming these faces to be 1. Each 2-face (square) is divided into two triangles, and the curvature matrices on these triangles resulting from our choice of connections turn out to be the same on this square and, besides, these differs from 1 only on the square built on two coordinate directions, not on diagonal(s).

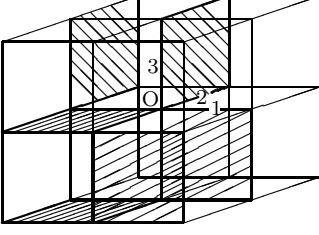


Figure 3: Periodic cell of the lattice with squares marked (shaded) in such a way that any edge does belong to one and only one of marked squares. It contains  $2 \times 2 \times 2$  elementary cells of the unmarked lattice.

Introduce some cubic notations and definitions. By  $\lambda$  we denote link in the coordinate direction  $\lambda$ ;  $\lambda, \mu, \nu, \rho, \dots = 1, 2, 3, 4$ . Let the coordinate 4 be  $t$ . By  $\mathcal{S}q$  denote a square. In particular,  $\mathcal{S}q = |\lambda\mu|$  means the square built on the coordinate directions  $\lambda, \mu$ . The connection matrix  $\Omega_\lambda$  is that one on the 3-cube built on the coordinates  $\mu, \nu, \rho$  (and also denoted as  $|\mu\nu\rho|$ ) complement to  $\lambda$ . The set  $\mathcal{M}$  for cubic decomposition of 3D leaf has periodic cell consisting of  $2 \times 2 \times 2$  elementary cubes, see fig.3 corresponding to

$$\mathcal{M} = \sum_{k_1, k_2, k_3} T_1^{2k_1} T_2^{2k_2} T_3^{2k_3} \left( |23| + \bar{T}_1|23| + \bar{T}_3|31| + \bar{T}_{12}|12| + \bar{T}_{23}|31| + \bar{T}_{123}|12| \right) \quad (26)$$

in the uppermost leaf. Here  $k_1, k_2, k_3$  are integers,  $T_\lambda$  is translation to the neighboring vertex in the direction  $\lambda$ . Expressions for the measure follow from those for the simplicial case (19) – (21) by replacing  $\sigma^2 \rightarrow \mathcal{S}q$ . There are several choices of the 4-cube containing a given square in the frame of which tensor of this square is defined. If the two area tensors are defined in the same frame, the result of integrating the measure over connections will depend on the scalar constructed of these two tensors. Therefore it seems to be a good idea to define area tensors in possibly different frames, as in fig.4, case (b). Of course, corresponding curvature matrices should be defined in the same frames. With this rule of definition the curvature matrices on the lateral squares of, say, the 3-prism with base  $|23|$  take the form  $\bar{T}_4^n R_{\mathcal{S}q}$ ,  $n = 1, 2, 3, \dots$ ,  $\mathcal{S}q = |42|, |43|, T_3|42|, T_2|43|$ ,

$$\begin{aligned} R_{|42|} &= \left( \bar{T}_3 \bar{\Omega}_1 \right) \left( \bar{T}_1 \Omega_3 \right) \Omega_1 \bar{\Omega}_3, \\ T_3 R_{|42|} &= \left( T_3 \bar{\Omega}_3 \right) \bar{\Omega}_1 \left( T_3 \bar{T}_1 \Omega_3 \right) (T_3 \Omega_1), \\ R_{|43|} &= \Omega_2 \bar{\Omega}_1 \left( \bar{T}_1 \bar{\Omega}_2 \right) \left( \bar{T}_2 \Omega_1 \right), \\ T_2 R_{|43|} &= \left( T_2 \bar{\Omega}_1 \right) \left( T_2 \bar{T}_1 \bar{\Omega}_2 \right) \Omega_1 (T_2 \Omega_2). \end{aligned} \quad (27)$$

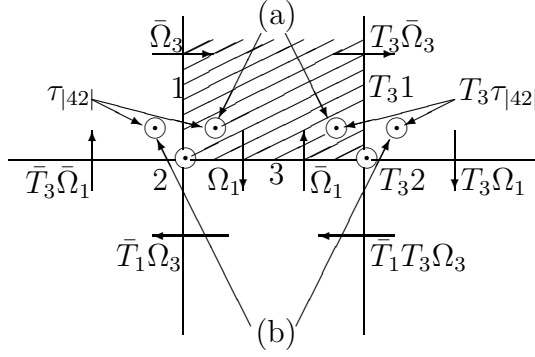


Figure 4: To assigning to the squares  $|42|$ ,  $T_3|42|$  the frames of definition of their area tensors  $\tau_{|42|}$ ,  $T_3\tau_{|42|}$ . In the pictured current 3D leaf these squares are observed as the links 2,  $T_3 2$ , respectively (shown perpendicular to the plane of picture). The chosen frame is pointed out by slightly parallel moving the given square to the chosen 4-cube (that is, parallel moving the given link to the chosen 3-cube in the pictured 3D leaf). (a).  $\tau_{|42|}$ ,  $T_3\tau_{|42|}$  are defined in the frame of the cube  $|123|$  (shaded). (b).  $\tau_{|42|}$ ,  $T_3\tau_{|42|}$  are defined outside the cube  $|123|$ .

(By default, notations  $\Omega_\lambda$ ,  $R_{|\lambda\mu|}$  are referred to the uppermost leaf.) Denote  $\Omega_4 \equiv U$ . The matrices  $\bar{T}_4^n \Omega_\alpha$ ,  $n = 1, 2, 3, \dots$ ,  $\alpha, \beta, \gamma, \dots = 1, 2, 3$  can be found in terms of  $\Omega_\alpha$ ,  $\bar{T}_4^k R_{|\alpha\beta|}$ ,  $\bar{T}_4^k U$ ,  $k = 0, 1, \dots, n-1$  from

$$R_{|23|} = \bar{U} (\bar{T}_4 \bar{\Omega}_1) (\bar{T}_1 U) \Omega_1, \quad \dots \text{cycle perm } 1, 2, 3 \dots \quad (28)$$

Thereby we find contribution of the  $t$ -like squares in terms of independent curvature matrices (on the leaf squares) and eventually matrices  $m_{\mathcal{S}q}$ .

For  $m_{|23|}$  the result reads

$$\begin{aligned} m_{|23|} = \pi_{|23|} + \sum_{n=1}^N \left( \prod_{k=0}^{n-1} \bar{T}_4^k \bar{R}_{|23|} \right) & \left\{ \left( \prod_{k=0}^{n-1} \bar{T}_4^k T_3 \bar{R}_{|12|} \right) (\bar{T}_4^n T_3 \tau_{|42|}) \left( \prod_{k=0}^{n-1} \bar{T}_4^k T_3 R_{|23|} \right) \right. \\ & \cdot \left( \prod_{k=0}^{n-1} \bar{T}_4^k T_3 \bar{T}_1 R_{|12|} \right) - \left( \prod_{k=0}^{n-1} \bar{T}_4^k R_{|12|} \right) (\bar{T}_4^n \tau_{|42|}) \left( \prod_{k=0}^{n-1} \bar{T}_4^k T_3 R_{|23|} \right) \left( \prod_{k=0}^{n-1} \bar{T}_4^k \bar{T}_1 \bar{R}_{|12|} \right) \\ & + \left( \prod_{k=0}^{n-1} \bar{T}_4^k R_{|31|} \right) (\bar{T}_4^n \tau_{|43|}) \left( \prod_{k=0}^{n-1} \bar{T}_4^k \bar{T}_2 R_{|23|} \right) \left( \prod_{k=0}^{n-1} \bar{T}_4^k \bar{T}_1 \bar{R}_{|31|} \right) \\ & \left. - \left( \prod_{k=0}^{n-1} \bar{T}_4^k T_2 \bar{R}_{|31|} \right) (\bar{T}_4^n T_2 \tau_{|43|}) \left( \prod_{k=0}^{n-1} \bar{T}_4^k T_2 R_{|23|} \right) \left( \prod_{k=0}^{n-1} \bar{T}_4^k T_2 \bar{T}_1 R_{|31|} \right) \right\} \end{aligned} \quad (29)$$

where  $N+1$  is the number of leaves, and the products of matrices are ordered according to the rule

$$\prod_{k=0}^n A_k = A_n A_{n-1} \dots A_0. \quad (30)$$

For other  $m_{\mathcal{S}q}$  we cyclically permute 1, 2, 3 and translate in the directions 1, 2, 3. For simplicity, here we have put equal to 1 the boundary values  $\Omega_\alpha$  on the upper leaf and to zero boundary values  $\pi_{\mathcal{S}q_0}$  on the lowest leaf. Besides that, gauge matrices  $\bar{T}_4^n U$ ,  $n = 0, 1, 2, \dots$  are set to be 1. This corresponds to extending the local frame from any 4-cube to the whole t-like 4-prism containing this 4-cube.

At the point  $\{\pm\zeta_{\mathcal{S}q} = 0, \pm\eta_{\mathcal{S}q} = 0 | \mathcal{S}q \notin \mathcal{M}\}$  where the factor in the measure corresponding to contribution from the squares  $\mathcal{S}q \notin \mathcal{M}$  reaches its maximum, and for uniform orthogonal lattice take  $\pm\pi_{|23|} = A \pm\Sigma_1/4$ ,  $\pm\tau_{|41|} = \pm\varepsilon \pm\Sigma_1/4$ ,  $\dots$  cycle permutations of 1, 2, 3  $\dots$ , then  $\pm R_{|23|} = \pm\Sigma_1$ ,  $\dots$ . In (29) we find sum of sign-altered terms so that  $m_{|23|} = \pi_{|23|} + O(\varepsilon)$ . For estimate, let  $A, \varepsilon$  be typical areas of the leaf and  $t$ -like squares, respectively. In the explicitly real expression for the measure (22) to be integrated, the cosine may become negative if for some variable  $\zeta$  or  $\eta$  we have  $\sinh \zeta = O(A/\varepsilon)$  or  $\sinh \eta = O(A/\varepsilon)$ . However, contribution from negative half-wave of cosine to the entire integral over  $\zeta, \eta$ -variables is dumped by the factor  $\exp(-O(A^2/\varepsilon))$  in this case. Therefore at  $A \geq A_0 = O(\sqrt{\varepsilon})$  contribution of the negative half-waves of cosine is dominated by positive ones, and resulting  $\pm\mathcal{N}$  is positive.

This is quite rough, sufficient estimate. In reality, the region of positivity of  $\pm\mathcal{N}$  well may be larger then this or even coincide with the whole range of varying the area tensors. The one-dimensional example is inequality  $\int_0^\infty f(x) \cos x dx > 0$  which can be easily proved to hold for *any* concave function ( $f''(x) > 0$ ; in particular, for  $f(x) = \exp(-kx)$  at *any*  $k > 0$ ). And even the inequality  $\pm\mathcal{N} > 0$  is, generally speaking, redundant for positivity means only  $+\mathcal{N} - \mathcal{N} > 0$ . Especially this circumstance is expected to promote the measure be positive when  $+\mathcal{N}$  and  $-\mathcal{N}$  are dependent. This takes place on the physical hypersurface singled out by the relations on area tensors of the type (6) which connect  $+v_{\sigma^2}$  and  $-v_{\sigma^2}$ .

Thus, completely discrete version of path integral in simplicial gravity can be naturally formulated with some boundary (initial/final) conditions. Representation of simplicial general relativity action in terms of area tensors and finite rotation matrices (connection and curvature) is used. Discrete connection and curvature on classical solutions of the equations of motion are not, strictly speaking, genuine connection and curvature, but more general quantities and, therefore, these do not appear as arguments of a function to be averaged, but are the integration (dummy) variables. Despite of unboundedness of general relativity action, path integral can be written in the form



resembling that with positive (real part of) effective action by moving integration contours over curvature to complex plane. This effective action is not purely real, but arguments are given that the resulting path integral measure is expected to be positively defined upon integrating over connection matrices. Up to some integrable factor, this measure is dominated by the product of exponentially (in area) falling off factors on separate areas.

It is interesting that our arguments use simplicial structure although built in a simple regular way of similar 3-dimensional leaves, but with rather complex structure of these leaves themselves; simplest leaf will not do. The work to extend the results to arbitrary structure is in order.

The present work was supported in part by the Russian Foundation for Basic Research through Grant No. 05-02-16627-a.

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